17

Dissertatio Astronomica
De
Methodo Inveniendi LATITUDINEM LOCI Ex observatis duabus Solis vel Stellæ cujusdam Altitudinibus.

Cujus

Particulam Priorem

Cons. Ampl. Facult. Philos. Aboëns.

Præside

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Tro-Tjenare och Fält-Camererare

Högädle

Herr CARL GUSTAF METHER,

Min Huldaste Fader,

Sällan skattar sig den tacksamme lyckeligare, än då han offenteligen får ådaga lägga sin tacksamhet; men ju lifligare dess känslor äro, desto mera fattas honom ord at dem rätteligen uttrycka. Denne förlägenhet drabbar ock mig, då jag vill nyttja tilfället af mitt första lärospåns utgifvande, at tolcka den brinnande erkänsla och vördnad, som Eder ömhet, Min Huldaste Fader, *och Edra välgärningar i mitt hjerta upptändt. Men blotta upsätet gäller hos Eder,* Min Huldaste Fader, *för värcket; uptagen fördenskull mitt ofullkomliga försök med Eder vanliga ynnest. — Den Högste förlänge Edra dagar! — Eder lefnad blifve lång och förnöjelig! Måtte en lycklig framtid lämna mig tilfälle at ådaga lägga den oförgängeliga vördnad hvarmed jag framhärdar at vara*

Min Huldaste Faders

lydigste Son
ANDERS JOHAN METHER.

§. I.

Latitudines locorum, quarum determinatio omnium fere obſervationum Aſtronomicarum baſis, & in univerſa Geographia atque re nautica maximi eſt momenti, diverſimode inveniri poſſe, ſcientia ſideralis edocet. Methodus quidem altitudinum Meridianarum maxime directa videtur & eo inprimis nomine commodiſſima, quod ex his ſine prolixo aliquo calculo Elevatio Poli ſeu Latitudo loci colligi queat. Obſervatio vero altitudinis Solis vel Stellæ cujusvis in meridiano quum ad certum reſtricta ſit tempus, haud raro accidit, ut vel nubilum cœlum vel aliæ circumſtantiæ eandem impediant. Quamobrem ad alias elevationem Poli inveniendi methodos ſæpius recurrendum eſt. Inter has inprimis attentionem meretur ea, qua ex obſervatis duabus quibuscunque Solis vel Stellæ cujusdam cognitæ altitudinibus, dato temporis intervallo inter utramque obſervationem, Elevatio Poli pro loco obſervationis ſeu Geographica hujus latitudo elicitur. Hinc ſpecimen Academicum edituri, operæ pretium duximus, hanc latitudinis inveniendæ rationem paullo pleniori ſubjicere diſquiſitioni. Nimiam vero prolixitatem ut evitemus, mox binas illas obſervationes ſecundum præcepta Aſtronomica debite correctas, adeoque datas pro utraque altitudines veras ſupponimus. Porro ſecundum revolutionem diurnam ſideris obſervati menſuratam poſtulamus tempus inter utramque obſervationem elapſum, unde huic reſpondens determinatur angulus horarius, qui videlicet eſt ad quatuor angulos rectos ſeu 360°, ut temporis intervallum inter binas illas obſervationes ad totum tempus diurnæ revolutionis ejusdem ſideris. Cognitam denique aſſumimus ſtellæ obſervatæ declinationem, eamque conſtantem, vel ſaltim tam parum variabilem, ut pro temporis ſpatio inter has obſervationes, ſine ſenſibili errore variationis iſtius ratio negligi queat. Problematis igitur, quo ex his datis latitudo loci inveſtigetur, varias adferre ſolutiones, ejusque in praxi Aſtronomiæ & præcipue in arte navigandi uſum examinare nobis jam propoſuimus.

§. II.

Inventu haud difficilis eſt directa Problematis noſtri ſolutio Trigonometrica. Si videlicet fuerit PZ (Fig. 1. 2.) arcus meridiani, in quo P polus æquatoris & Z zenith loci, ſintque pro duabus iſtis obſervationibus loca Solis vel Stellæ obſervatæ A & B, ductis arcubus circulorum maximorum AZ, BZ, AP, BP & AB, erit AP = BP = complemento datæ declinationis, AZ & BZ diſtantiæ a vertice ſeu datarum

alti-

altitudinum complementa, quorum sit AZ $<$ BZ, APB angulus horarius tempori inter observationes elapso respondens (§. 1.), & PZ complementum latitudinis quaesitae. Primo igitur in Δ APB ex datis duobus lateribus AP, BP, & angulo intercepto APB, investigentur reliqui anguli & tertium latus. Et quidem ob AP $=$ BP, facillima fiet resolutio Trianguli APB; si videlicet bisecetur ang. APB arcu PM, hic etiam bisecabit basin AB & huic normaliter insistet, unde Δ APM ad M rectangulum erit, adeoque (assumto Sinu Toto $=$ I)

$$Cotg.\, PAB = Cos.\, AP.\, Tg.\, \tfrac{1}{2}\, APB$$

$$\&\ Sin\, \tfrac{1}{2}\, AB = sin\, AP.\, sin\, \tfrac{1}{2}\, APB$$

quarum formularum ope inveniuntur AB $=$ 2 AM & $\angle$ PAB. Ex invento vero AB datisque praeterea AZ & BZ in Δ AZB investigatur angulus ZAB per formulam:

$$Sin\, \tfrac{1}{2} ZAB = \sqrt{\frac{Sin\, \tfrac{1}{2}(ZB - AZ + AB).\, sin\, \tfrac{1}{2}(BZ + AZ - AB)}{Sin\, AB.\, Sin\, AZ.}}$$

Cognitis sic angulis ZAB & PAB obtinetur $\angle$ ZAP. Huic vero angulo duplex valor competit, prout vel ad unam vel ad alteram partem ipsius AB sumantur arcus AZ & BZ. Problema scilicet nostrum generaliter sumtum quadraticum esse, adeoque duplicem admittere solutionem, ope Schematis facillime perspicitur. Si namque Polis A & B per Z duo describantur circuli; (nisi fuerit $\angle$ ZAB vel $=$ 0 vel $=$ 180°) manifestum est hos circulos praeter Z in alio quodam

 pun-

puncto se invicem secare, quod verticem determinabit alterius loci, in quo eædem observationes obtinere possunt. In praxi vero quum semper facillime discerni queat, ad utram ipsius Zenith partem cadat circulus per A & B transiens, nulla hinc metuenda erit ambiguitas in valore anguli ZAP ex inventis ZAB & PAB colligendo. Hoc autem invento, datisque lateribus AZ & AP eundem comprehendentibus in Δ PZA invenitur latus tertium. Ducto scilicet ex Z ad AP arcu perpendiculari ZN, in Δ ZNA ad N rectangulo erit

$$Tg.\,AN = Tg.\,ZA.\,Cos.\,ZAP$$

unde AN, adeoque etiam PN innotescunt. His denique cognitis, secundum notissimam Regulam Trigonometricam infertur:

$$Cos.\,ZP = \frac{Cos.\,AZ.\,Cos.\,PN}{Cos.\,AN} = Sin.\ \text{Latitudinis}$$

quæsitæ.

Exempli caussa si institutis alicubi duabus observationibus, intervallo temporis $= I^h$ inter se distantibus, datæ sint altitudines Solis veræ 17° 13′ & 19° 41′, existente declinatione ejus australi 20°; adeoque $\angle$ APB = 15° seu APM = 7° 30′, BZ = 72° 47′, AZ = 70° 19′ & (designante P polum borealem) AP = 110°; elevatio poli pro loco observationis ita computabitur:

$L\ Cos\ AP = \bar{1}.5340517$	$L\ Sin\ AP = \bar{1}.9729858$
$L.\ Tg.\ \frac{1}{2}\ APB = \bar{1}.1194291$	$L\ Sin\ \frac{1}{2} APB = \bar{1}.1156977$
$L.\ Cotg.\ PAB = \bar{2}.6534808$	$L.\ Sin\ \frac{1}{2}\ AB = \bar{1}.0886835$

$PAB = 92^{\circ}\ 34'\ 41'',4$ | $L\,Sin\,\frac{1}{2}(BZ - AZ + AB) = \overline{1}.7583255$

$\frac{1}{2}\,AB = 7^{\circ}\ 2'\ 43'',2$ | $L\,Sin\,\frac{1}{2}(BZ + AZ - AB) = \overline{1}.9555051$

$AB = 14^{\circ}\ 5'\ 25'',4$ | $-\ L\,Sin\ AB = 0.6135777$

$\frac{1}{2}(BZ + AZ) = 71^{\circ}\ 33'\ 00'',0$ | $-\ L\,Sin\ AZ = 0.0261481$

$\frac{1}{2}(BZ - AZ) = 1^{\circ}\ 14'\ 00'',0$ | $2\ L\,Sin\ \frac{1}{2}\,ZAB = \overline{1}.7535564$

$\frac{1}{2}(BZ - AZ + AB) = 8^{\circ}\ 16'\ 43'',2$ | $L\,Sin\ \frac{1}{2}\,ZAB = \overline{1}.8767782$

$\frac{1}{2}(BZ + AZ - AB) = 64^{\circ}\ 30'\ 16'',8$

$\frac{1}{2}\,ZAB = 48^{\circ}\ 50'\ 51'',5$

$ZAB = 97^{\circ}\ 41'\ 43'',0$

Prout igitur in loco obſervationis culminatio Solis aut auſtralis eſt aut borealis, angulus PAZ erit aut $= ZAB - PAB = 5^{\circ}\ 7'\ 1'',6$ aut $= 360^{\circ} - ZAB - PAB = 169^{\circ}\ 43'\ 35'',6$. Pro utroque caſu calculus ita ſe habet:

Caſ. 1.	*Caſ. 2.*
$ZAP = 5^{\circ}\ 7'\ 1'',6$	$ZAP = 169^{\circ}\ 43'\ 35'',6$
$L\,Coſ.\,ZAP = \overline{1}.9982657$	$L\,Coſ.\,ZAP = \overline{1}.9929809$
$L\,Tg.\,AZ = 0.4464523$	$L\,Tg.\,AZ = 0.4464523$
$L\,Tg.\,AN = 0.4447180$	$L\,Tg.\,AN = 0.4394332$
$AN = 70^{\circ}\ 14'\ 38'',4$	$AN = 109^{\circ}\ 58'\ 43'',9$
$PN = 39^{\circ}\ 45'\ 21'',6$	$PN = 00\ 1'\ 16'',1$
$-\ L\,Coſ.\,AN = 0.4710637$	$-\ L\,Coſ.\,AN = 0.4663898$
$L\,Coſ.\,PN = \overline{1}.8857992$	$L\,Coſ.\,PN = 0.0000000$
$L\,Coſ.\,AZ = \overline{1}.5273997$	$L\,Coſ.\,AZ = \overline{1}.5273997$
$L\,Sin\ Latit. = \overline{1}.8842626$	$L\,Sin\ Latit. = \overline{1}.9937895$
$Latit. = 50^{\circ}\ 0'\ 4'',9$ Bor	$Latit. = 80^{\circ}\ 20'\ 0''$ Auſtr.

 Schol.

Schol. 1. Hinc ſimul pro utravis obſervatione computari poteſt tempus verum & angulus azimuthalis. Inventis ſcilicet angulo PAZ & ſegmentis AN, PN, ſecundum formulam

$$Tg.\ ZPA = \frac{Tg.\ PAZ.\ Sin\ AN}{Sin\ PN}$$

invenitur angulus ZPA, unde ob datum angulum APB ſimul innoteſcit ang. ZPB. Utrique horum angulorum reſpondens (§. 1.) tempus a tempore culminationis ſtellæ obſervatæ ſubtractum vel huic additum, prout ſcilicet vel ante vel poſt culminationem inſtituta fuerit obſervatio, dabit tempus verum quæſitum. Angulus vero azimuthalis vel poſitio meridiani reſpectu verticalium ZA vel ZB determinari poterit per regulam:

$$Sin\ PZA = \frac{Sin\ ZPA.\ Sin\ PA}{Sin\ AZ}\ vel\ Sin\ PZB = \frac{Sin\ ZPB.\ Sin\ PB}{Sin\ BZ}$$

Sic in Exemplo allato pro utroque caſu & tempus verum & azimuth ſequenti calculo eruitur:

Caſ. 1.	*Caſ.* 2.
L *Tg.PAZ* $= \bar{2}.\ 9520590$	L. *Tg.PAZ* $= \bar{1}.\ 2582234$
L *Sin AN* $= \bar{1}.\ 9736545$	L *Sin AN* $= \bar{1}.\ 9731126$
--L *Sin PN* $= 0.\ 1941463$	--L *Sin PN* $= 3.\ 4330439$
L *Tg.ZPA* $= \bar{1}.\ 1198598$	L *Tg ZPA* $= 2.\ 6643799$
ZPA $= 7^{o}\ 30'\ 26'',\ 5$	*ZPA* $= 90^{o}\ 7'\ 26'',\ 8$
L *Sin ZPA* $= \bar{1}.\ 1161228$	L *Sin ZPA* $= \bar{1}.\ 9999990$
L *Sin PA* $= \bar{1}.\ 9729858$	L *Sin PA* $= \bar{1}.\ 9729858$
--L *Sin AZ* $= 0.\ 0261481$	--L *Sin AZ* $= 0.\ 0261481$
L *Sin PZA* $= \bar{1}.\ 1152567$	L *Sin PZA* $= \bar{1}.\ 9991329$
PZA $= 172^{o}\ 30'\ 30''$	*PZA* $= 93^{o}\ 37'\ 10''$

Angulo horario ZPA respondet in Cas. 1. tempus 30′ 1″, 8 & in Cas. 2:do $6^h\ 0'\ 29''$, 8. Quod si igitur ante meridiem instituta fuerit illa Solis in A Observatio, tempus verum huic competens erit in casu priori $11^h\ 29'58''$,2 & in posteriori $5^h\ 59'\ 30''$, 2 a. m.

Schol. 2. Generatim etiam, & quidem calculo paurum prolixiori, definiri potest latitudo loci datis punctorum observatorum A & B declinationibus utcunque inaequalibus; unde ex observatis altitudinibus duarum stellarum diversarum, quarum cognitae sunt declinationes & ascensiones rectae, elevatio Poli computari potest. Primo scilicet ex intervallo temporis inter utramque observationem & differentia Ascensionum rectarum facile investigatur angulus APB, ex quo porro & lateribus eundem comprehendentibus AP, BP (quae sunt datarum declinationum complementa) in Δ APB per praecepta Trigonometrica inveniuntur ang. PAB & latus AB. Quo facto, sicut supra resolvendo Δ Δ ABZ & ZAP obtinetur ZP. = complemento quaesitae latitudinis.

Schol. 3. Problematis inversi, quo data latitudine loci, ex observatis duabus stellae cujusdam altitudinibus declinatio hujus investigatur, similis omnino est resolutio. Si videlicet PZ referat circulum declinationis in quo sit P polus & Z stella observata, sintque A & B loca ipsius zenith pro duabus istis observationibus; dabitur angulus horarius APB tempo-

ri

ri inter utramque observationem elapso respondens (§. 1.) unde & ex datis AP = BP = elevationi æquatoris, nec non datarum altitudinum complementis AZ & BZ computari potest ZP complementum declinationis quæsitæ.

§. III.

Problema nostrum Algebraice solvendi sequens nobis commodissima videtur methodus. Sit elevatio Poli quæsita $= y$, declinatio stellæ observatæ $= D$ (quam versus polum elevatum numeratam supponimus, adeoque negative sumendam si sidus observatum versus polum depressum declinet), angulus horarius intervallo temporis inter utramque observationem respondens $= 2m$, datarum stellæ a zenith distantiarum semisumma $= a$ earundemque semidifferentia $= b$ & angulus horarius respondens intervallo inter culminationem stellæ & momentum temporis medium inter binas istas observationes $= z$. Designante igitur (Fig. 1. 2.) P polum, Z zenith loci cujus latitudo quæritur, A & B loca stellæ in binis istis observationibus, & ductis per hæc puncta arcubus circulorum maximorum, nec non arcu PM bisecante angulum APB; erit PZ $= 90^{\circ} - y$; AP $=$ BP $= 90^{\circ} - D$, ZB $= a + b$, ZA $= a - b$, $\angle$ APM $= \angle$ BPM $= m$ & $\angle$ ZPM $= z$, adeoque ZPB $= z + m$ & ZPA $= \pm z \mp m$. Jam vero (Elem. Trig. Sphær.) in Δ ZAP est Cos. ZA =

Sin

Sin ZP Sin AP. Cos. ZPA + Cos. ZP. Cos. AP, hoc est
$Cos(a-b) = Cos y\, Cos D\, Cos(z-m) + Sin y\, Sin D$ (I).

Pari ratione in Δ ZPB habetur
$Cos(a+b) = Cos y\, Cos D\, Cos(z+m) + Sin y\, Sin D$ (II).

Unde si æquatio II. subtrahatur ab æquatione I, ob $Cos p - Cos q = 2 Sin \frac{1}{2}(q+p) Sin \frac{1}{2}(q-p)$ erit
$Sin\, a\, Sin\, b = Cos y\, Cos D\, Sin\, m\, Sin\, z$ (III).

Si vero eædem æquationes addantur, ob $Cos p + Cos q = 2 Cos \frac{1}{2}(q+p) Cos \frac{1}{2}(q-p)$ erit
$Cos a\, Cos b = Cos y\, Cos D\, Cos m\, Cos z + Sin y\, Sin D$, vel
$Cos a\, Cos b - Sin y\, Sin D = Cos y\, Cos D\, Cos m\, Cos z$ (IV).

Si porro æquatio III. per $Cos m$ multiplicetur & æquatio IV per $Sin m$, oriuntur æquationes
$Sin a\, Sin b\, Cos m = Cos y\, Cos D\, Sin m\, Cos m\, Sin z$ (V) &
$Cos a\, Cos b\, Sin m - Sin y\, Sin D\, Sin m = Cos y\, Cos D\, Sin m\, Cos m\, Cos z$ (VI.). Singulorum membrorum in utraque æquatione V. & VI, si sumantur quadrata, & hinc emergentes æquationes addantur, ob $Sin z^2 + Cos z^2 = 1$ exterminatur z, & pro $Cos y^2$ substituendo $1 - Sin y^2$ obtinetur æquatio unicam quantitatem incognitam scilicet $Sin y$ involvens:
$Sin a^2\, Sin b^2\, Cos m^2 + Cos a^2\, Cos b^2\, Sin m^2 - 2 Cos a\, Cos b\, Sin m^2\, Sin D\, Sin y + Sin m^2\, Sin D^2\, Sin y^2 = Cos D^2\, Sin m^2\, Cos m^2 - Cos D^2\, Sin m^2\, Cos m^2\, Sin y^2$ (VII). Terminis hujus æquationis VII debite dispositis & divisis per $Sin m^2$, facta-

 que

que substitutione $Cosm^2 CosD^2 + SinD^2 = 1 - Sinm^2 CosD^2$ prodit æquatio:

$$(1 - Sinm^2 CosD^2) Siny^2 - 2\, Cosa\, Cosb\, SinD\, Siny =$$
$$= CosD^2\, Cosm^2 - Sina^2\, Sinb^2\, Cotg.\, m^2 - Cosa^2\, Cos.\, b^2,$$

unde denique, (compendii caussa ponendo $1 - Sinm^2 CosD^2 = \lambda$ atque $Cosm^2 CosD^2 - Sina^2 Sinb^2 Cotgm^2 = \mu$) secundum vulgarem æquationum quadraticarum methodum eruitur:

$$Siny = \frac{Cosa\, Cosb\, SinD \pm \sqrt{\lambda\mu - Cosa^2\, Cosb^2\, Cosm^2\, CosD^2.}}{\lambda}$$

Hæc quidem formula maxime directam atque generalem problematis nostri solutionem præbet; in praxi tamen ob calculum nimis prolixum minus commode adhiberi potest. Præcipuus vero ejus usus est ad indolem problematis accuratius intelligendam, variosque hujus casus dijudicandos, quam ob rem eam adferre voluimus.

§. IV.

Ad praxin utilissimi hujus Problematis faciliorem reddendam, nimiamque calculi prolixitatem evitandam, de compendiosiori ejusdem solutione invenienda solliciti fuerunt Astronomi. Circa annum 1740. Cl. CORNELIUS DOUWES, munere tunc temporis functus Examinatoris Officialium Maritimorum atque nautarum in Collegio Archithalassorum Amstelodamensi, indirectam proposuit problema nostrum solvendi methothodum. Quum videlicet ubique conjectura saltim assequi liceat latitudinem loci a vera non multum aberrantem, adeo ut de paucorum tantum scrupulorum correctione quæstio plerumque sit; laudatus hic Auctor modum ostendit, quo aliquot tabularum ope latitudo loci vera ex supposita, facili calculo erui poterit. Hæc inventio tanti æstimata fuit, ut ab his, quibus methodorum longitudines inveniendi perficiendarum cura Londini Anglorum commissa est (*The Commissioners of Longitude*) præmio 50 Libr. Sterling. condecoraretur. Tabulas has regulasque pro earundem applicatione videre licet in *Tables requisite to be used with the Nautical Ephemeris* Edit. 2. ut & in pluribus Scientiæ nauticæ compendiis recentioribus. Ex nostratibus Cl. CHIERLIN ad calcem libri: *Sjömåns Dagelige Assistent*, Holm. 1777 easdem adfert ex RICH. HARRISONII *Logarithme Solar Tables* depromtas.

Ipsam quidem Analysin *Douwesianam* videre nobis non licuit. Ex inspectis vero Tabulis regulisque nominatis eadem facile colligi potest. Si videlicet fuerit latitudo loci supposita $= p$ & quæsita seu vera y (adeo ut differentia $y - p$ sit satis exigua), declinatio stellæ observatæ $= D$, altitudinum datarum major $= \alpha$ & minor $= \beta$, tempus inter utramque observationem $= 2h$, nec non intervallum temporis inter culminationem stellæ & medium inter has observationes momentum elapsi $= t$, assumta una hora seu 24:ta parte revolutionis diurnæ ejusdem ſideris pro unitate, erunt anguli horarii ipſis h & t reſpondentes $15°h$ & $15°t$. Eodem igitur modo ac in § præc. demonſtrabitur eſſe

$$Sin\,\alpha = Cos\,y\,Cos\,D\,Cos\,15°(t-h) + Sin\,y\,Sin\,D \text{ (I.) \&}$$

$$Sin\,\beta = Cos\,y\,Cos\,D\,Cos.\,15°(t+h) + Sin\,y\,Sin\,D \text{ (II.)}$$

Æquationem II ab æqu. I. ſubtrahendo ob $Cos\,15°(t-h) - Cos\,15°(t+h) = 2\,Sin\,15°h\,Sin\,15°t$, erit

$$Sin\,\alpha - Sin\,\beta = 2\,Cos\,y\,Cos\,D\,Sin\,15°h\,Sin\,15°t \text{ (III.)}$$

In hac æquatione ob differentiam $y - p$ admodum parvam, pro $Cos\,y$ ſubſtitui poteſt $Cos\,p$, quamobrem erit quam proxime:

$$2\,Sin\,15°t = \frac{Sin\,\alpha - Sin\,\beta}{Sin\,15°h\,Cos\,p\,Cos\,D} \text{ (IV.)}$$

Unde innoteſcit t, adeoque etiam $t-h$ ſeu intervallum temporis inter obſervatam altitudinem majorem & tranſitum ſtellæ per meridianum. Porro ob $Cos\,15°(t-h) = 1 - 2\left(Sin\,15°\,\frac{t-h}{2}\right)^2$ & $Cos\,y\,Cos\,D + Sin\,y\,Sin\,D = Cos\,(y-D)$ æqu. I ita transformari poteſt:

$$Cos\,(y-D) = Sin\,\alpha + 2\,Cos\,y\,Cos\,D\left(Sin\,\frac{15°(t-h)}{2}\right)^2 \text{ (V)}$$

Si jam in ultimo termino hujus æquationis pro $Cos\,y$ ſubſtituatur eidem quam proxime æqualis $Cos\,p$, & pro $t-h$ adhibeatur valor ejus jam inventus, poſita altitudine ſtellæ obſervatæ meridiana $=M$, ob $y-D = 90° - M$ erit

Sin

$Sin M = Sin \alpha + 2\left(Sin 15^\circ \frac{t-h}{2}\right)^2 Cosp\, CosD$ (VI)

Si igitur statuatur $\frac{1}{Cosp\ CosD} = G$; $\frac{1}{Sin 15^\circ h} = H$; $2\, Sin 15^\circ t = T$ & $2\left(Sin 15^\circ \frac{t-h}{2}\right) = S$; patet facile construi posse tabulas in quibus pro singulis valoribus ipsorum h & t inveniuntur $LogH$, $LogT$ & $LogS$. In casu quovis si præterea computetur $LogG = 2\, Log\, Rad. - Log\, Cosp - Log\, Cosd$, vel $Log\, G = L\, Secp\, SecD - 2\, L\, Rad$; erit per æquat IV.

$$LogT = LogG + LogH + Log\,(Sin\,\alpha - Sin\,\beta).$$

Huic $LogT$ respondens tempus $= t$ in tabula constructa invenitur, unde simul innotescit tempus $t - h$, cui respondens ex Tabula depromatur $LogS$. Si porro $\frac{S}{G}$ seu $S. Cosp\ CosD$, dicatur N; adeoque

$$LogN = LogS - LogG;$$

Inventus hic numerus N ipsi $Sin\,\alpha$ additus dabit per æqu. VI

$$Sin M = Sin\,\alpha + N$$

Unde cognita fit altitudo meridiana M, adeoque invenitur $y = 90^\circ - M + D$

Schol. 1. In tabulis secundum methodum *Douwesianam* constructis assumi solet Rad = 100000 ejusque Logarithmus = 5, neque in iis ultra partes 100000:mas extendi solent Logarithmi.

Schol. 2. Quoties latitudo inuenta a supposita notabiliter admodum differt, repetito opus est calcu-

 lo,

lo, pro p assumendo valorem ipsius y nuperrime inventum. Ita repetita operatione donec differentia illa fiat satis exigua, vera tandem obtinetur latitudo quaesita.

Illustrationis caussa secundum hanc Methodum supputatum adferre lubet exemplum in § II. allatum, existente scil. $D = -20°$; $\alpha = 19° 41'$; $\beta = 17° 13'$ & $h = 30'$, ponendoque praeterea $p = 50° 40'$ B.

$p = 50° 40'$	$Log\ Rad - Log Cos p = 0.19803$
$D = -20°$	$Log\ Rad - Log Cos D = 0.02701$
$\alpha = 19° 41'$	$Log\ G = 0.22504$
$\beta = 17° 13'$	$Log(Sin \alpha - Sin \beta) = 3.61098$
$Sin\ \alpha = 33682$	$Log\ H = 0.88430$
$Sin\ \beta = 29599$	$Log\ T = 4.72032$
$Sin\ \alpha - Sin\ \beta = 4083$	$Log\ S = 2.95599$
$h = 0° 30'$	$Log\ G = 0.22504$
$t = 1^h\ 0'\ 50''$	$Log\ N = 2.73095$
$t - h = 0^h\ 30'\ 50''$	$M = 20° 1'$
$N = 538$	$90° - M = 69° 59'$
$Sin\ \alpha = 33682$	$D = -20°$
$Sin\ M = 34220$	$y = 49° 59'$ B

In hoc igitur exemplo ob majorem differentiam $p - y = 41'$ repetendus erit calculus ponendo $p = 49° 59'$

$Log\ Rad - Log\ Cos p = 0.19178$
$Log\ Rad - Log\ Cos D = 0.02701$
$Log\ G = 0.21879$

$$\text{Log } G = 0.21879$$
$$\text{Log }(\text{Sin } \alpha - \text{Sin } \beta) = 3.61098$$
$$\text{Log } H = 0.88430$$
$$\text{Log } T = 4.71407$$
$$\text{Log } S = 2.93223$$
$$\text{Log } G = 0.21879$$
$$\text{Log } N = 2.71344$$

$$h = 0^h\ 30'$$
$$t = 1^h\ 0'$$
$$t - h = 0^h\ 30'$$
$$\text{Sin } \alpha = 33682$$
$$N = 517$$
$$\text{Sin } M = 34199,\ M = 20^\circ\ 0'$$
$$90^\circ - M = 70^\circ\ 0'$$
$$D = -\ 20^\circ\ 0'$$
$$y = 50^\circ\ 0'\ B$$

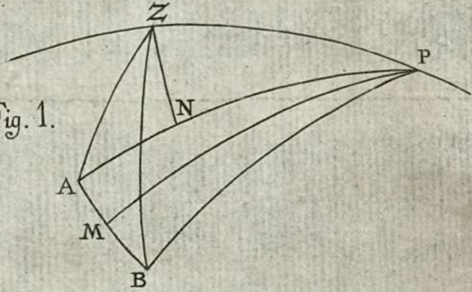
Post hanc vero repetitionem quum $y - p$ sit nonnisi $=$ $1'$, inventa latitudo $50^\circ\ 0'$ satis exacta censeri potest, quod etjam calculo supra (§. 11) secundum directam methodum instituto comprobatur.

§. V.

Ex formulis §. præc. allatis haud difficile est perspectu, etjam sine tabulis istis *Douwesianis*, eadem methodo vulgarium tabularum Trigonometricarum ope commode satis problema nostrum solvi posse, si modo pro tempore inter observationes dato sumatur (§. 1.) huic respondens angulus horarius. Neque hæc temporis in angulum conversio tantæ est difficultatis, ut ob eam evitandam ad peculiares quasdam Tabulas recurrere opus sit. Maximum vero in Methodo *Douwesiana* incommodum nobis parere videtur usus, quem postulat, sinuum naturalium, qui videlicet & prolixiorem reddunt calculum, & in tabulis nautarum usui destinatis desiderantur. Hoc autem levi facta substitutione evitatur. Dicatur scilicet angulus horarius tempori inter observationes elapso respondens $2m$, (adeo ut sit $m = 15^\circ h$ §. IV). & ponatur

Cosp

Fig. 1.



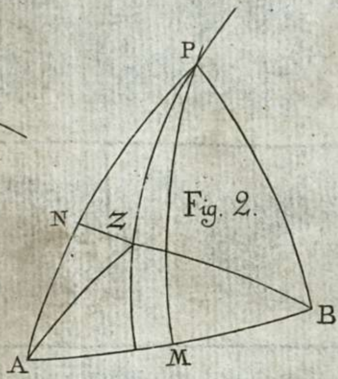


Fig. 2.

C: L: Schultz sculp: